LIMITS ON THE STRENGTH OF EUROPA'S ICY SHELL FROM TOPOGRAPHIC SPECTRA.

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Introduction: Radio echo sounding on Europa may provide estimates of the depth to the brittle-ductile transition. This information should be consistent with the topography spectrum. Here we derive an upper bound on the amplitude and shape of the topographic spectrum that can be supported by the strength of the lithosphere. Topographic features that exceed this upper bound must be supported by local compensation.

Lithospheric Strength: *Byerlee* [1] showed that the frictional resistance on a fault τ depends on the fault normal stress σ_n and the coefficient of friction and is largely independent of material properties. Suppose that the brittle layer of ice has closed fractures oriented in all directions and a differential stress $(\Delta\sigma = \sigma_1 - \sigma_3)$ is applied. Frictional sliding will occur on optimally-oriented fractures when the resolved shear stress exceeds the Byerlee criteria. The magnitude of this differential stress is called the yield strength [2] and it is primarily a function of the overburden pressure as follows:

$\Delta\sigma = -3\rho gz$ (horizontal compression)

$\Delta\sigma = 0.7\rho gz$ (horizontal extension)

For lack of additional information about the ductile layer, we'll assume it has a linear decrease in strength with depth and zero strength is achieved at twice the depth of the brittle ductile transition (Figure 1). One can now calculate the maximum bending moment that can be maintained by the lithosphere before it fails M_s. The saturation bending moment is the integral of the yield strength over depth times the distance to the brittle/ductile transition

$$M_s = \int_0^{2z_b} \Delta\sigma(z)(z - z_b)dz = 0.6\rho gz_b^3$$

where ρ is the density (1000 kg m^{-3}) and g is the acceleration of gravity (1.3 m s^{-2}). The saturation bending moment increases as the cube of the depth to the brittle/ductile transition.

Topographic Moment: This estimate of saturation bending moment provides an upper bound on the amplitude of the topography that is supported by stress in the shell. The vertical load of positive topography must compensated by a nearby negative topographic load. This high to low dipole must be maintained by the strength of the lithosphere. Consider sinusoidal topography of wavelength λ and amplitude w_o. The moment that must be applied at the origin to maintain the topography is given by the following formula.

$$M = g\rho \int_{-\lambda/4}^{\lambda/4} w_o \sin\left(\frac{2\pi x}{\lambda}\right)x dx = \frac{g\rho w_o \lambda^2}{2\pi^2}$$

This topographic moment must be less than the saturation bending moment that can be supported by the strength of the plate M_s. Equating these two moments provides an upper bound on the amplitude of the topography as a function of the characteristic wavelength of the topography.

$$w_o < \frac{2\pi^2 M_s}{g\rho\lambda^2} \cong \frac{\pi^2 z_b^3}{\lambda^2}$$

We see that for a given saturation moment, the maximum amplitude of the topography decreases rapidly with increasing wavelength. The nice feature of this formula is that it contains measurable quantities - the wavelength of the topography and the depth to the brittle/ductile transition. An additional bound on the topographic spectrum is that topographic slopes cannot exceed the angle of repose (~30°). This can be translated into a constraint on topographic amplitude versus wavelength.

$$w_o < \frac{0.57\lambda}{2\pi}$$

Overall the amplitude of the topography must be less than the smaller of the two limiting mechanisms (Figure 2).

Conclusions:
- Saturation bending moment depends on the cube of the depth to the brittle-ductile transition.
- The maximum topography that can be maintained by the strength of the lithosphere decreases as the square of the wavelength of the topography.
- Significant amplitude (>100 m) topography on Europa with wavelength greater than 100 km cannot be supported by lithospheric strength and therefore must be supported by a local compensation mechanism.

References:
[1] Byerlee, J. D. Friction of rocks. *Pageoph* **116**, 615-626 (1978). [2] Brace, W. F. & Kohlstedt, D. L., *J. Geophys. Res.* **85**, 6248-6252 (1980). [3] Prockter, L.M., et al., *J. Geophys. Res.,* 107(E5), 5028, 2002.
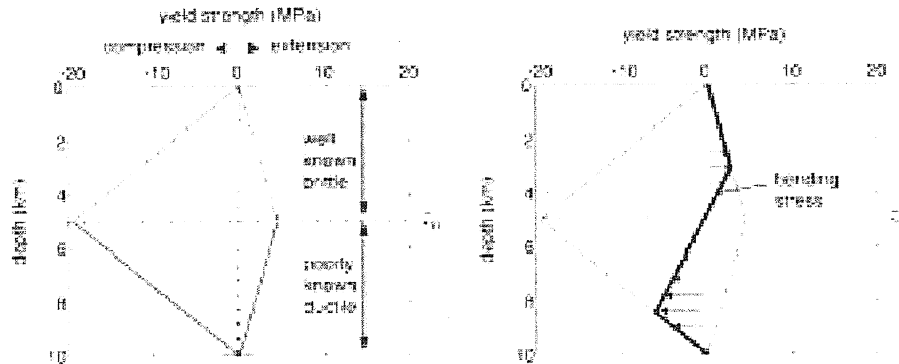

Figure 1. Strength of the lithosphere as a function of depth in tension and compression. (left) Stress is relieved by the lowest-strength deformation mechanism resulting in a yield strength envelope. The strength of the upper brittle layer follows Byerlee's Law while the strength of the lower ductile layer is largely unknown. (right) Concave-up bending of the lithosphere will cause extension in the upper half of the plate and compression in the lower half of the plate.
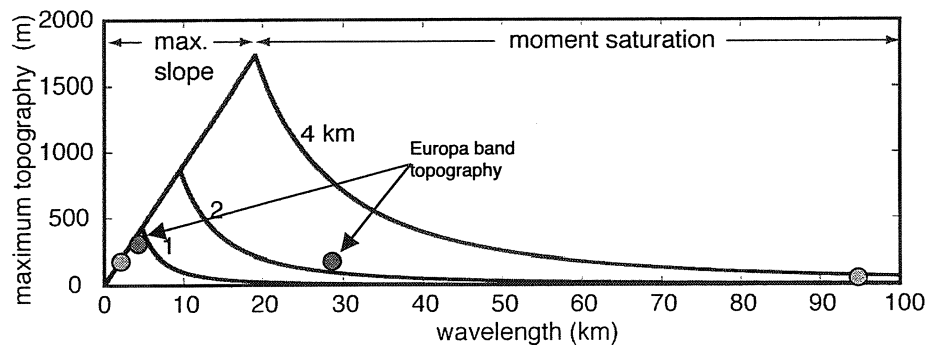

Figure 2. Maximum amplitude topography that can maintained entirely by the strength of an icy shell for brittle/ductile transition depths of 1 km, 2 km and 4 km. The amplitude of the short-wavelength topography is limited by the angle of repose (~30°). The amplitude of the longer wavelength topography is limited by the strength of the lithosphere. The red dots mark the typical amplitude/wavelength of banded topography on Europa [3]. The green dots mark approximate requirements for a new mission.